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ON THRUST ESTIMATES FOR FLAPPING FOILS

K. Streitlien*
The Levich Institute, The City College of New York, New York, New York 10031, U.S.A.

AND

G. S. Triantafyllou
The Levich Institute & Department of Mechanical Engineering, The City College of New York New York, New York 10031, U.S.A.

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The average thrust developed by flapping foils seen among the birds and fast swimming marine animals is often estimated experimentally from the time-averaged velocity profile in the wake. We present a parametric investigation of the error due to neglect of the unsteady terms, under the assumption of two-dimensional ideal flow. A numerical simuation algorithm allows comparison of exact average thrust and mean-flow momentum flux in the wake, and two simple models for the wake, based on linear foil theory and the Kármán vortex street, provide further insight into the problem. The main observation is that mean-flow wake surveys tend to overestimate the thrust force, most severely for high reduced frequencies and low values of the proportional feathering parameter. We also find that a thrust estimate based on the Kármán vortex street model yields surprisingly accurate results over the entire range of tested parameters. The linear theory predicts thrust well for all but the highest Strouhal number.

1. INTRODUCTION

It is well known that a foil oscillating in heave and pitch can produce a mean forward thrust force, T, through generation of a reversed vortex street (von Kármán & Burgess 1935, p. 308). This is the principle by which many swimming and flying animals propel themselves, and it also covers a special mode of hovering seen among insects and small birds. When averaged over one or more periods of motion in time, the vortex street has the velocity profile of a jet. In experimental work, it is customary to estimate the average thrust based on the momentum flux, Q, associated with this profile, (Koochesfahani 1989; Freymuth 1990; Anderson 1996). It is the purpose of this paper to assess the error that results from disregarding unsteady contributions in such mean-flow estimates.

In this investigation we employ a computer program that assumes unsteady, two-dimensional ideal flow (Streitlien & Triantafyllou 1995). Although animal flows are inherently three-dimensional and may be strongly influenced by viscous effects, these basic

*Currently at: The Autonomous Underwater Vehicles Laboratory, Massachusetts Institue of Technology Sea Grant Program, Cambridge, MA 02139, U.S.A.

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assumptions will expose the parametric relationship between the foil motion and thrust estimates. The numerical model steps the flow solution forward in time by resolving it into two components: potential flow around a Joukowski foil in arbitrary motion (found by conformal mapping from a cylinder) and flow induced by vortices that are shed at the trailing edge. A new vortex is released every time step, such that the Kutta condition and Kelvin's theorem are satisfied, and these vortices form the propulsive wake of the foil as they convect downstream. The circle theorem ensures that the body boundary condition is satisfied as well. Note that we take the word "wake" to mean any downstream velocity anomaly, including jets.

We will consider simple harmonic motion of the foil (of chord c) in a free stream, U, from left to right. The pitch angle is positive clockwise and given by

$$\theta = \theta_0 \cos(\omega t - \psi). \tag{1}$$

The pitch axis is at a fixed streamwise location, $x = b$ measured from midchord, and oscillates in the direction transverse to the oncoming flow to produce the desired heaving motion,

$$h = h_0 \cos(\omega t). \tag{2}$$

The simulation procedure admits the calculation of instantaneous force and moment, which can be averaged over one oscillation period. Several periods after the simulation starts, these averaged forces reach steady state values; zero for lift and moment, and when the motion parameters are right, a positive value for thrust. We shall consider this to be the true mean thrust force, and denote it by T_s (the subscript s indicating simulation).

Figure 1 shows a typical snapshot of the program output at the very last time step, with the downstream portion of the wake rolled up into a reversed vortex street. Also shown are wake velocity profiles in a reference frame in the fluid at rest, $\bar{u}(y)$, obtained by averaging over the last period. The thrust based on the momentum flux of this mean flow is given by†

$$Q_s = \int_{-\infty}^{\infty} (U + \bar{u})\bar{u}\,\mathrm{d}y. \tag{3}$$

This expression would be exact for steady flows far downstream where pressure has reached ambient value. Equation (3) is often used as an approximate value for mean thrust in experimental work, because the cost and complexity of instrumenting the moving foil with pressure or force transducers can be prohibitive.

The mean velocity profiles in Figure 1 change with the downstream distance in the wake. They have somewhat unrealistic features at intermediate distance, presumably due to the lack of viscous diffusion in the model. However, in most case Q_s is constant from about 20 chord-lengths down, and this is the value we take in the following analysis.

2. MODELS

Two simple models for thrust and its mean-flow estimate can be derived from (a) the theory of the von Kármán vortex street (von Kármán & Burgess 1935) and (b) linear nonuniform aerofoil theory (von Kármán & Sears 1938; Sears 1941; Lighthill 1970). These theories make

†All force expressions are for unit span and unit fluid density.

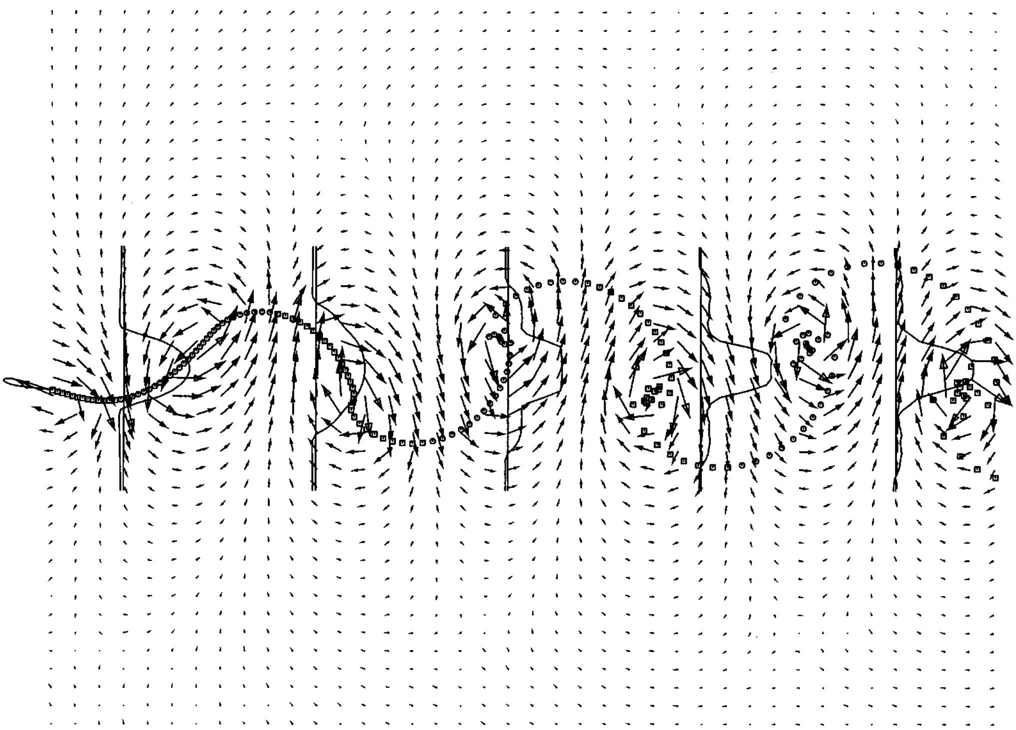

Figure 1. Program output at the last time step of a simulation. The small dots indicate the location of the individual vortices, and the vectors show the instantaneous velocity. The profiles show streamwise velocity after averaging over the preceding period.

different assumptions about the velocity field in the wake, and provide an interesting supplement to the computer simulations.

(a) *The Kármán vortex street* (indicated by subscript k) is characterized by the distance between its two staggered rows of point vortices, d, the streamwise separation between vortices in each row, l, and each vortex circulation Γ. The average drag force associated with a Kármán vortex street, including unsteady terms, was derived by von Kármán & Burgess (1935; pp. 346–349). Their analysis can be carried through unchanged with the sign of Γ reversed, leading to the average thrust force

$$T_k = \frac{\Gamma d}{l}\left(U + \frac{\Gamma}{l}\tanh\frac{\pi d}{l}\right) - \frac{\Gamma^2}{2\pi l}. \tag{4}$$

The mean velocity profile has a "top hat" shape, as if the vorticity were distributed into shear layers of zero thickness:

$$\bar{u} = \begin{cases} 0, & |y| > d/2, \\ \Gamma/l, & |y| < d/2. \end{cases} \tag{5}$$

Thus, if the wake behind the flapping foil had the form of a Kármán vortex street, equation (3) would predict the following thrust:

$$Q_k = d(U + \Gamma/l)\Gamma/l; \tag{6}$$

and the thrust predicted on the basis of mean velocity profile would be overestimated by an amount

$$Q_k - T_k = \frac{\Gamma^2}{l}\left[\frac{d}{l} - \frac{d}{l}\tanh\frac{\pi d}{l} + \frac{1}{2\pi}\right]. \tag{7}$$

The expression in square brackets is bounded between 0·16 and 0·25 for all d/l.

The values for Γ, l and d in this model are determined from the numerical simulation, by grouping together the discrete vortices in the wake that have the same sign and that were shed during the same half-cycle. By evaluating the zeroth and first moments (i.e. the circulation and centroid) of each group we obtain a street of "meta-vortices" whose characteristics at the final simulation step can be inserted in equations (4) and (6).

(b) *Linear theory* (indicated by subscript l) assumes that motion amplitudes are small, and in this limit the vertical displacement of a point x on the foil is

$$y = [h_0 + \theta_0 e^{-i\psi}(x - b)]e^{i\omega t}, \tag{8}$$

where the factor $e^{i\omega t}$ implies that the real part should be taken. As Lighthill (1970) points out, the heave-pitch phase angle, ψ, can be set to $\pi/2$ without loss of generality, resulting in the average thrust

$$\begin{aligned} T_l = \frac{\pi c}{2}&\left|\left[\omega\theta_0\left(b - \frac{c}{4}\right) + i(U\theta_0 - \omega h_0)\right]C(\sigma/2) + \frac{\omega\theta_0 c}{4}\right|^2 \\ &- \frac{U\pi c\theta_0}{2}\mathscr{Im}\left\{\left[\omega\theta_0\left(b - \frac{c}{4}\right) + i(U\theta_0 - \omega h_0)\right]C(\sigma/2)\right\} \\ &- \frac{\pi(c\omega\theta_0)^2 b}{8}. \end{aligned} \tag{9}$$

Here, $\sigma = \omega c/U$ is the reduced frequency and C is the Theodorsen function, formed by modified Bessel functions of the second kind,

$$C(\sigma/2) = \frac{K_1(i\sigma/2)}{K_0(i\sigma/2) + K_1(i\sigma/2)}. \tag{10}$$

The formulation of von Kármán & Sears (1938) yields, in addition to the lift and moment, the strength of the trailing vortex sheet

$$\gamma(x, t) = \frac{-2\pi[\omega\theta_0(b - c/4) - i(\omega h_0 - U\theta_0)]}{K_0(i\sigma/2) + K_1(i\sigma/2)}\, e^{i\omega(t - x/U)}. \tag{11}$$

We assume as a first-order approximation that the vortex sheet far downstream remains on the path of the trailing edge:

$$y(x, t) = [h_0 - i\theta_0(c/2 - b)]e^{i\omega(t - (x - c/2)/U)}. \tag{12}$$

It can be shown that a vortex sheet with sinusoidally varying strength and displacement, if assumed to be of infinite extent, has an average velocity profile in the shape of half an ellipse.

The momentum flux is found to be

$$Q_l = U^2 A \left[\pi + \frac{16A}{3|h_0 - \mathrm{i}\theta_0(c/2 - b)|} \right], \tag{13}$$

where A is the real part of

$$\frac{[\omega\theta_0(b - c/4) - \mathrm{i}(\omega h_0 - U\theta_0)]\mathrm{e}^{-\mathrm{i}\sigma}[h_0 + \mathrm{i}\theta_0(c/2 - b)]}{U(\mathrm{K}_0(\mathrm{i}\sigma/2) + \mathrm{K}_1(\mathrm{i}\sigma/2))}. \tag{14}$$

Thus, we can compute the thrust, T, and its mean-flow estimate, Q, in three different ways that can be compared.

3. PARAMETERS

For the purposes of this investigation, we take $\psi = \pi/2$ and $b = -c/4$ to be constant and we employ a 12% thick, symmetric Joukowski profile. The remaining parameter space can be spanned in many different ways; here we choose the following nondimensional groups:

(i) The proportional feathering parameter:

$$\Theta = \frac{\theta_0 U}{\omega h_0}; \tag{15}$$

(ii) The Strouhal number for the pitch axis motion:

$$\mathrm{St} = \frac{h_0 \omega}{\pi U}; \tag{16}$$

(iii) The reduced frequency:

$$\sigma = \frac{\omega c}{U}. \tag{17}$$

The significance of these parameters can be understood in terms of the path that the pitch axis traces out as the foil moves from right to left in the fluid at rest. The proportional feathering parameter identifies the degree to which the foil pitch angle coincides with the slope of this path. The Strouhal number is proportional to the maximum slope of the path, and can be considered a measure of the nonlinearity of the motion. Finally, the reduced frequency is a measure of the chord-length compared to the path wavelength.

For convenience, we take $U = c = 1$, which make the various Ts and Qs nondimensional coefficients. Our plan is to vary each one of the nondimensional numbers around a basic case, thus marking out a cross in parameter space and hopefully exposing the most important trends. The range of variation that can be specified in each parameter is limited by the wake dynamics. For some combinations, vortex street instability precludes any meaningful time averaging, while for other cases (high St, for instance), more than two distinct vortices form during each period, making the simple wake model invalid. The basic case is the one shown in Figure 1, which has $\Theta = 0{\cdot}5$, $\mathrm{St} = 1/(2\pi)$, $\sigma = 1$. This combination of motion parameters yields fair efficiency§ ($\eta = 0{\cdot}75$), and allows some flexibility in

§ Efficiency is the ratio of useful work, UT, to the input power required for the heave and pitch oscillation.

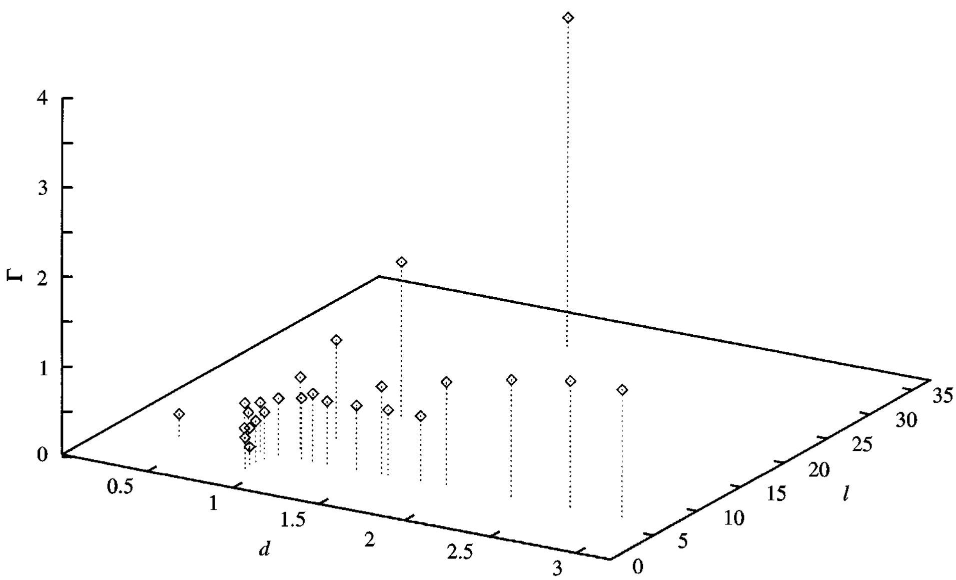

Figure 2. Coverage of the vortex street parameter space.

changing the parameters while still generating a sustained vortex street. We are also able to include St $\approx 0{\cdot}3$ which was predicted by Triantafyllou *et al.* (1993) to be optimal, based on linear instability theory for the mean wake.

In order to verify that we have produced a representative selection of wake configurations, Figure 2 shows the variations in Γ, l and d resulting from all the runs to be presented.

4. RESULTS

First, the effect of the proportional feathering parameter is considered, by changing the pitch amplitude while holding the heave amplitude and period fixed. Figure 3 shows that the highest thrust is obtained for low Θ, on account of the large leading edge suction, while approaching negative values at feathering parameters near unity. The thrust estimates based on the vortex street model, T_k, and linear theory, T_l agree well with the exact mean thrust over the whole range, with the former being consistently around 0·01 too low. Thrust computed from momentum flux of the mean flow, Q_s, gives accurate predictions for $\Theta > 0{\cdot}4$, but is about 45% too high at zero pitch. The vortex street model, Q_k, shows the same trend, but indicates that the discrepancy between exact thrust and mean-flow estimate is larger than it really is. The linear mean-flow momentum flux, Q_l, has a slightly smaller slope, predicting more uniform error.

Next, the Strouhal number dependence is studied, by varying the heave and pitch amplitudes at the same rate (thus keeping Θ constant). Figure 4 shows that the linear responses go like St^2, but the simulated results all reach nonlinear saturation at the higher Strouhal numbers. Again the mean-flow momentum flux, Q_s, overpredicts the thrust, although by less than 8% over the entire range shown. For the von Kármán vortex street model, the comments of the previous case apply: the unsteady vortex street analysis is remarkably accurate, whereas the averaged flow carries too much momentum.

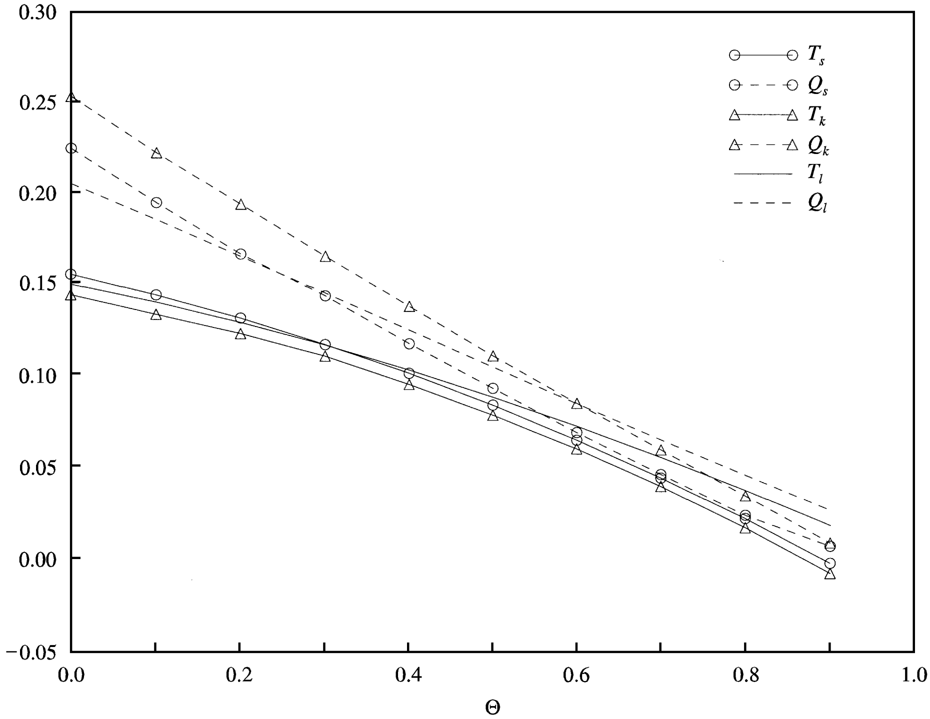

Figure 3. Trust and its estimates as functions of the proportional feathering parameter.

Finally, the effects of reduced frequency have been isolated by varying the frequency and heave amplitude in inverse proportion, keeping pitch amplitude constant. In Figure 5 we see that the difference between the mean-flow momentum flux and the correct unsteady calculation becomes serious as the reduced frequency increases beyond unity. This is reflected in the vortex street model as well, but not so clearly in the linear theory, which predicts a rather constant overestimate at all σ. Thrust itself, however, is predicted very well by linear theory for $\sigma < 1$ and is accurate to within 10% up to $\sigma = 2$. T_k also has the largest error at $\sigma = 2$, being about 10% below T_s.

5. DISCUSSION AND CONCLUSIONS

From this limited investigation of the parameter space for flapping foils we can make the following conclusions.

Simulation, the vortex street model, and linear theory all agree: mean-flow momentum flux overestimates thrust.

The error is small for reduced frequencies below 1·0 and proportional feathering parameters above 0·4. In a real flow, viscous and turbulent diffusion smoothes out the wake velocity field and will likely reduce the discrepancy. Consequently this should be taken as a conservative conclusion.

The thrust predicted by the simple unsteady von Kármán vortex street model (T_k) is surprisingly accurate, underestimating the thrust force by no more than 10% in all cases

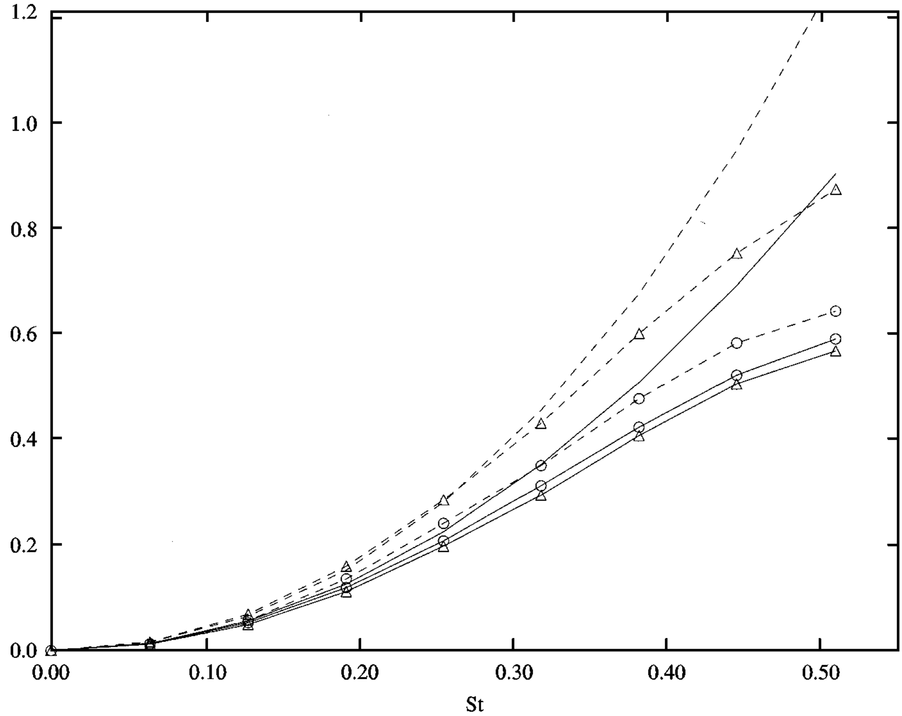

Figure 4. Strouhal number dependence in thrust and its estimates. Legend as for Figure 3.

studied. However, we should keep in mind that the vortex street model may not be straightforward to employ in the evaluation of actual experiments. First, unless quantitative data is obtained for the entire flow field by methods such as particle-image velocimetry, it is not possible to determine Γ directly, although it may be possible to estimate it from the mean-flow data combined with traditional visualization techniques. Second, experimental wakes are subject to inevitable three-dimensional flow components and instabilities, as well as viscous effects. Anderson (1996) reports that these can make it difficult to locate the quantify more than a few rolled up vortices, with a resulting uncertainty in the vortex street parameters.

Although it shows the correct trends, the relative discrepancy in mean-flow thrust estimate, Q/T, is overestimated by the vortex street model. The excess in Q/T is not totally unexpected, as the Kármán vortex street is in a sense the most unsteady wake one could imagine, with all vorticity singularly concentrated into points. Our original motivation was to investigate whether Q/T could be predicted by the simple models and used to correct experimental results, but this cannot be recommended. We can only identify certain regions of parameter space where wake survey thrust calculations should be interpreted with care.

The Strouhal number is a good indicator of the applicability of the linear theory. Above $\mathrm{St} = 0{\cdot}4$, thrust, T, becomes seriously overestimated. Along the other parameter axes, linear theory accurately predicts thrust. Linear theory mean-flow momentum flux, Q_l, is not reliably related to the other thrust estimates; presumably, the sinusoidal vortex sheet is not a very realistic model for the wake for downstream.

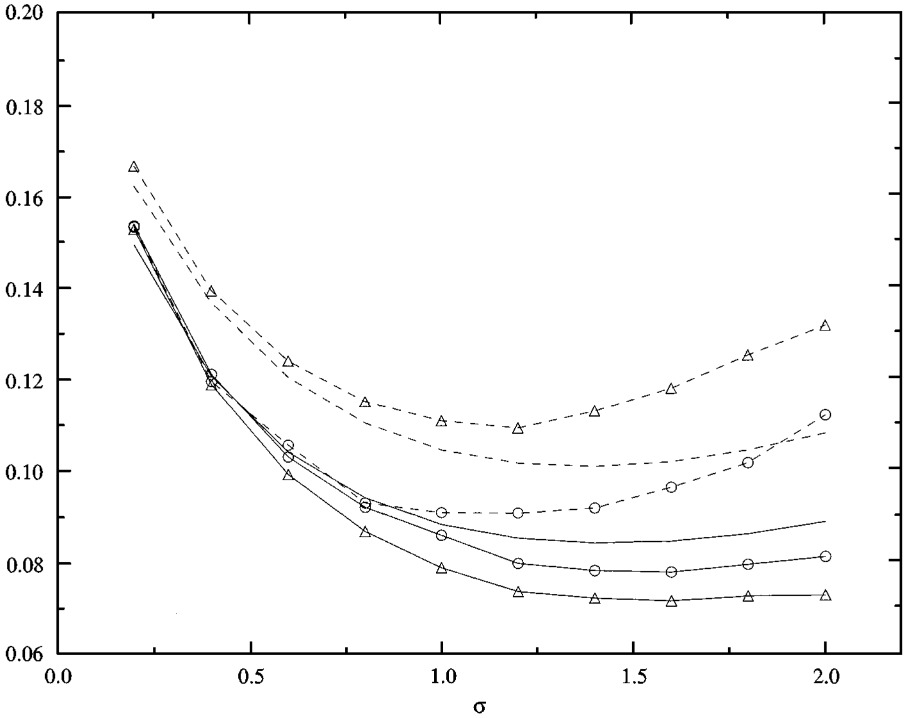

Figure 5. Reduced-frequency dependence in thrust and its estimates Legend as for Figure 3.

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